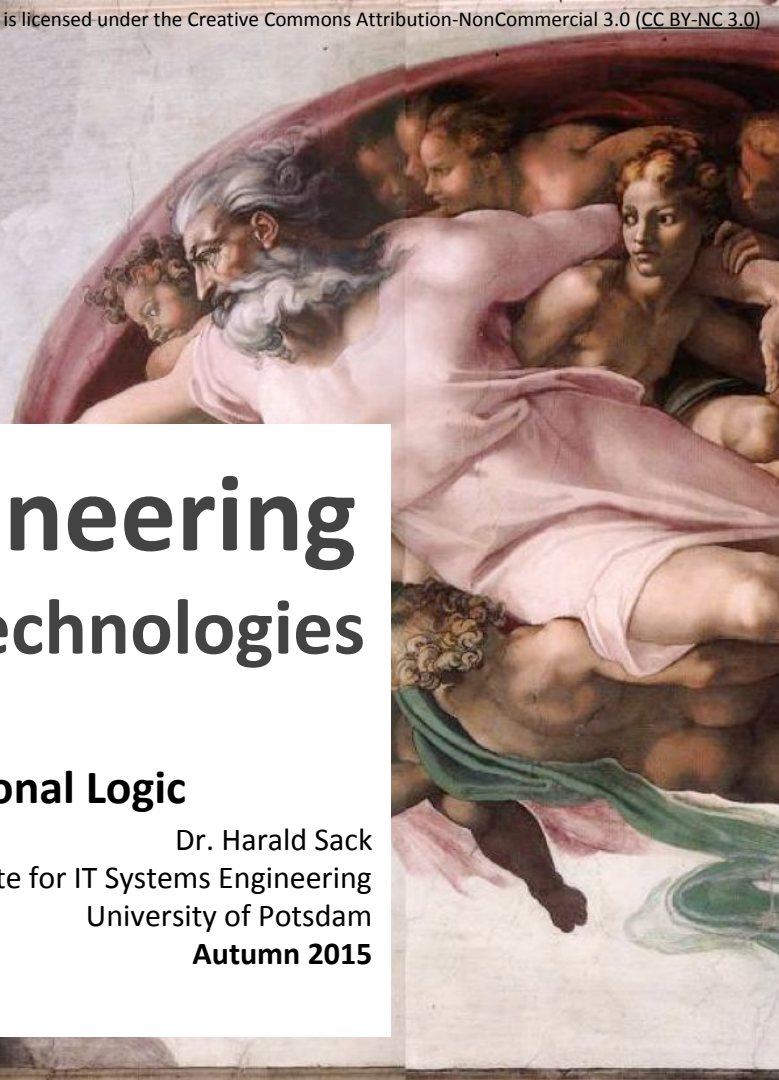
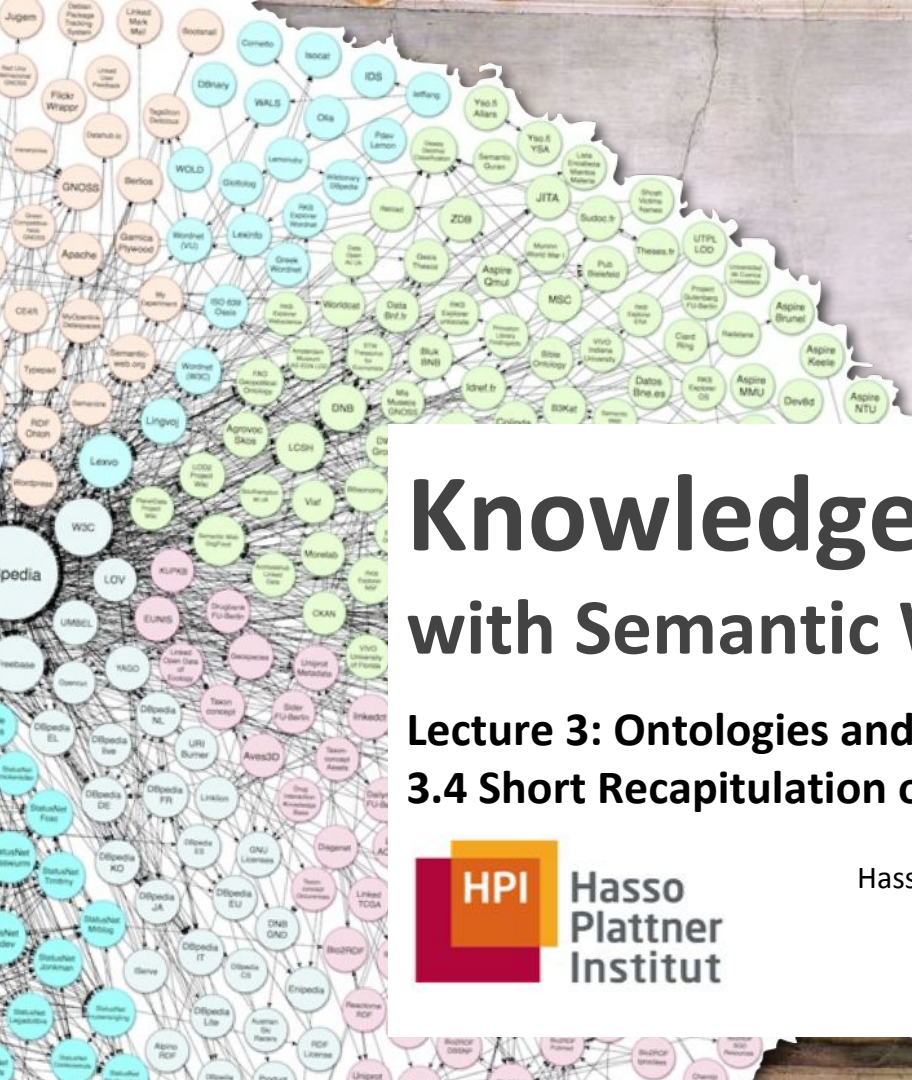




Knowledge Engineering with Semantic Web Technologies

Lecture 3: Ontologies and Logic

3.4 Short Recapitulation of Propositional Logic



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Autumn 2015

Propositional Logic - PL

- in propositional logic the world consists simply of **facts** and nothing else (**statements of assertions**)
- Example for propositional logic assertions and deductions:
 - If it rains, the road will get wet.
 - If the moon is made out of green cheese, then cows can fly.
 - If Oliver is in love, then he will be happy.
- The world consists out of objects and properties that distinguish one object from another.
- Between objects are relations. Some relations are unique, i.e. functions.

Propositional Logic - PL

Syntax:

- **Logical connectives:** $Op = \{ \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,) \}$,
- a set of symbols Σ
- with $\Sigma \cap Op = \emptyset$ and $\{\text{true}, \text{false}\}$
- **Production rules** for propositional formulae (propositions):
 - all atomic formulas are propositions
(*all elements of Σ*)
 - if ϕ is a proposition, then also $\neg\phi$
 - if ϕ and ψ are propositions,
then also $\phi \wedge \psi, \phi \vee \psi, \phi \rightarrow \psi, \phi \leftrightarrow \psi$
- **Priority:** $\neg$ prior to $\wedge, \vee$ prior to $\rightarrow, \leftrightarrow$

connective	name	intentional meaning
$\neg$	negation	“not”
$\wedge$	conjunction	“and”
$\vee$	disjunction	“or”
$\rightarrow$	implication	“if - then”
$\leftrightarrow$	equivalence	“if, and only if, then”

Propositional Logic

How to model facts?

Simple Assertion	Modeling
The Moon is made of green cheese	g
It rains	r
The street is getting wet	n

Composed Assertion	Modeling
if it rains, then the street will get wet.	$r \rightarrow n$
If it rains and the street does not get wet, then the moon is made of green cheese.	$(r \wedge \neg n) \rightarrow g$

Propositional Logic

Model-theoretic Semantics

- **Interpretation I:**

Mapping of all atomic propositions to $\{t, f\}$.

- If $\mathcal{F}$ is a formula and $\mathcal{I}$ an Interpretation, then $\mathcal{I}(\mathcal{F})$ is a truth value computed from $\mathcal{F}$ and $\mathcal{I}$ via **truth tables**.

$\mathcal{I}(p)$	$\mathcal{I}(q)$	$\mathcal{I}(\neg p)$	$\mathcal{I}(p \vee q)$	$\mathcal{I}(p \wedge q)$	$\mathcal{I}(p \rightarrow q)$	$\mathcal{I}(p \leftrightarrow q)$
f	f	t	f	f	t	t
f	t	t	t	f	t	f
t	f	f	t	f	f	f
t	t	f	t	t	t	t

Propositional Logic

Model-theoretic Semantics

- We write $\mathcal{I} \models \mathbf{F}$, if $\mathcal{I}(\mathbf{F}) = \mathbf{t}$,
and call Interpretation $\mathcal{I}$ a **Model** of formula $\mathbf{F}$.
- Rules of Semantics:
 - $\mathcal{I}$ is model of $\neg\varphi$, iff $\mathcal{I}$ is not a model of φ
 - $\mathcal{I}$ is model of $(\varphi \wedge \psi)$, iff $\mathcal{I}$ is a model of φ AND of ψ
 - ...
- Basic concepts:
 - tautology
 - satisfiable
 - refutable
 - unsatisfiable (contradiction)

Propositional Logic

Some Properties of PL

- **Decidability**

- All **true entailments** can be found, and all **false entailments** can be refuted, as long as you spent enough time.
- $\Rightarrow$ there always exist terminating automatic theorem prover

- Another useful property:

- $\{\varphi_1, \dots, \varphi_n\} \models \varphi$ holds, iff semantic entailment/inference
- $(\varphi_1 \wedge \dots \wedge \varphi_n) \rightarrow \varphi$ is a tautology syntactic entailment/inference
- The decision, if an assertion is an tautology, can be made via truth tables
- In principle this equals the evaluation of all possible interpretations



05: Short Recapitulation of First Order Logic

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